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SOLUTION BY THE PROPOSER.

Let w be the specific weight of the fluid; then the atmospheric pressure is wh_0. Let a be the inner radius of the cone and P = the vertical component of the pressure between the fluid and the cone.

The weight of the fluid is $\frac{1}{3}\pi a^2hw$, and
The weight of the cone is $\frac{1}{3}\pi a^2h\sigma w[(1+x)^3-1]$.
The pressure between the horizontal surface and the cone is πa^2hw.
The atmospheric pressure upon the cone has the vertical component $\pi a^2(1+x)^2wh_0$.
From the equilibrium of the fluid,

$$P+\tfrac{1}{3}\pi a^2hw=\pi a^2hw.$$

From the equilibrium of the cone,

$$\tfrac{1}{3}\pi a^2h\sigma w[(1+x)^3-1]+\pi a^2(1+x)^2wh_0 \nless P=\tfrac{2}{3}\pi a^2hw.$$

Hence, for equilibrium,

$$\frac{h_0}{h}(1+x)^2+\sigma x(1+x+x^2/3)\nless 2/3.$$

2733 [1918, 444]. Proposed by J. L. RILEY, Stephenville, Texas.

An ellipse of constant eccentricity passes through the focus of a parabola and has its foci on the curve. Find the envelopes of its axes.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let F be the focus, $(a, 0)$ of the parabola,

$$y^2=4ax;\tag{1}$$

F_1, F_2, the foci, (x_1, y_1), (x_2, y_2) of the ellipse in any one of its positions; $2A$, e, the major axis and constant eccentricity.

We have

$$FF_1+FF_2=2A,\tag{2}$$

$FF_1=x_1+a$, and $FF_2=x_2+a$; and hence

$$2a+x_1+x_2=2A.\tag{3}$$

Let the equation of the major axis of the ellipse be

$$y=mx+b.\tag{4}$$

Eliminate y from (4) and (1), and we have the quadratic giving the abscissas x_1, x_2 of F_1, F_2,

$$m^2x^2-(4a-2bm)x+b^2=0\tag{5}$$

and so

$$x_1+x_2=(4a-2bm)/m^2.\tag{6}$$

Similarly eliminating x,

$$my^2-4ay+4ab=0,\tag{7}$$

and then

$$y_1+y_2=4a/m,\tag{8}$$

and (3) becomes

$$(2am^2-2bm+4a)/m^2=2A.\tag{9}$$

Again,

$$(F_1F_2)^2=(x_1-x_2)^2+(y_1-y_2)^2=4A^2e^2.\tag{10}$$

From (5),

$$x_1x_2=b^2/m^2,\tag{11}$$

and from (7),

$$y_1y_2=4ab/m.\tag{12}$$

From (6) and (11),

$$(x_1-x_2)^2=16a(a-bm)/m^4,\tag{13}$$

and from (8) and (12).

$$(y_1-y_2)^2=16a(a-bm)/m^2.\tag{14}$$

Substituting (13) and (14) in (10),

$$16a(1+m^2)(a-bm)/m^4 = 4A^2e^2. \tag{15}$$

From (15) and (9) we obtain:

$$\frac{4a(1+m^2)(a-bm)}{\{a(1+m^2)+a-bm\}^2} = e^2. \tag{16}$$

An accommodating feature of this problem is the analytical form of the left member of (16). Let

$$a - bm = a(1+m^2)\tan^2\epsilon; \tag{17}$$

then (16) becomes

$$\frac{2\tan\epsilon}{1+\tan^2\epsilon} = 2\sin\epsilon\cos\epsilon = \sin 2\epsilon = e,$$

consistent with e not > 1 for the ellipse. Substituting b from (17) in (4), the equation of the major axis is the quadratic in m,

$$(x - a\tan^2\epsilon)m^2 - ym + a\,(1-\tan^2\epsilon) = 0. \tag{18}$$

The condition for equal values of m in (18) is

$$y^2 = 4a(1-\tan^2\epsilon)(x - a\tan^2\epsilon),$$

the envelope of the major axis.

The equation of the straight line through the center of the ellipse and perpendicular to (4) is

$$y - \frac{2a}{m} = -\frac{1}{m}\left(x - \frac{2a-bm}{m^2}\right);$$

or, substituting b from (17) and arranging,

$$ym^3 + (x - 2a - a\tan^2\epsilon)m^2 - a\sec^2\epsilon = 0. \tag{19}$$

Differentiating with respect to the variable parameter m, we find $m = -2(x-2a-a\tan^2\epsilon)/3y$ and (19) becomes

$$y^2 = \frac{4}{27a\sec^2\epsilon}(x - a - a\sec^2\epsilon)^3,$$

the envelope of the minor axis of the ellipse.

Also solved by the PROPOSER.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

Assistant Professor A. F. CARPENTER, of the University of Seattle, has been promoted to an associate professorship of mathematics.

At the University of California, Associate Professor C. A. NOBLE has been promoted to a full professorship of mathematics, and Assistant Professor B. M. WOODS to a full professorship of aërodynamics.

Mr. E. D. MEACHAM, of the University of Oklahoma, has been promoted to an assistant professorship of mathematics.

Assistant Professor H. H. CONWELL, of the University of Idaho, has been promoted to an associate professorship of mathematics.